“Wasted” Animals

As described in the parent entry, Button et al.(2013a) followed the standard hypothesis testing paradigm and were concerned with “detecting” an effect, meaning finding a P-value ≤ 0.05. If we follow this framework in which all that matters is whether the study detects an effect, then we can also follow Button et al. in terming all of a study’s animals as “wasted” if the study has $P>0.05$. This is what they mainly focused on, but they also acknowledged the problem of excessive sample size, so another source of “wasted” animals is those beyond what would have been needed to find $P\leq 0.05$.

Letting P_N denote the power with sample size N, we can analyze how the combination of these two sources of “waste” changes as sample size increases.

Going from N to N+1 will increase the number of wasted animals by one if we already have $P\leq 0.05$ at sample size N. The increase in the expected number of wasted animals due to excess sample size is therefore
P_N.

The change in the expected number of wasted animals due to not reaching $P\leq 0.05$ is this expected number with N+1 minus this expected number with N, which is simply
$(1-P_{N+1})(N+1) - (1-P_N)N$.

In the case that we have $P\leq 0.05$ with N and $P>0.05$ with N+1, there is one animal that is counted in both the above expressions. The amount of this double counting is bounded above by $1-P_{N+1}$, because this is the probability of $P>0.05$ with N+1, and we must also have $P\leq 0.05$ with N for the double counting to occur. So the net effect on the expected number of wasted animals is greater than the sum of the above two terms minus this bound:
$P_N + (1-P_{N+1})(N+1) - (1-P_N)N - (1-P_{N+1})$
$= P_N + N + 1 - P_{N+1}(N+1) - N + P_N N - 1 + P_{N+1}$
$= P_N - P_{N+1}(N+1) + P_N N + P_{N+1}$
$= (N+1)P_N - NP_{N+1}$

This quantity is generally positive due to diminishing marginal returns, which can be shown as follows.

Bacchetti, et al. (2005, Figure and Appendix) showed that under general conditions we have decreasing power per subject as sample size increases, so
$P_N/N > P_{N+1}/(N+1)$, which implies
$(N+1)P_N > NP_{N+1}$, which implies
$(N+1)P_N - NP_{N+1} > 0$, as needed.

Thus, increasing sample size increases the expected number of “wasted” animals.